Via evaluation of the Lyapunov exponent, we report the discovery of three pairs of phase space regimes of quasi-periodic orbits of charged particles trapped in a dipole magnetic field. Besides the low energy regimes that have been studied extensively and cover more than 10% in each dimension of the phase space, there are two pairs of high energy regimes, the largest of which covers more than 4% in each dimension of the phase space. Particles in these high energy orbits may explain some anomalies observed in space and may be realized in plasma experiments on the Earth.

It is well-known that there are quasi-periodic orbits around stable periodic orbits in Hamiltonian systems with 2 degrees of freedom. Since periodic orbits appear to have a negligible measure in the phase space, they are difficult to realize in nature. Quasi-periodic orbits, on the other hand, may have a finite volume in the 4 dimensional phase space and be readily detectable.

Since a chaotic orbit has at least one positive Lyapunov exponent, the maximum Lyapunov exponent of quasi-periodic orbits then should be zero. Via calculation of the Lyapunov exponent of orbits of charged particles in a dipole magnetic field, we scanned the corresponding phase space and found several regimes of quasi-periodic orbits associated with stable periodic orbits in the equatorial plane. Our numerical results also suggest a continuous spectrum of these orbits from stable periodic, to quasi-periodic, and eventually to chaotic ones.